

On the paper "On Green's relations, 2^0 -regularity and Quasi-ideals in Γ -semigroups"

Niovi Kehayopulu

Department of Mathematics, University of Athens

157 84 Panepistimiopolis, Athens, Greece

email: nkehayop@math.uoa.gr

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This is about the paper in the title published in by Hila and Dine in [1]. According to Lemmas 2.13 and 2.14 of this paper a Γ -semigroup M is left (resp. right) regular if and only if $a\mathcal{L}(a\gamma a)$ (resp. $a\mathcal{R}(a\gamma a)$) for all $a \in M$ and all $\gamma \in \Gamma$. The Theorem 2.15, Corollary 2.16, Proposition 2.17, Proposition 2.21, Theorem 2.22, Corollary 2.23, Theorem 2.24, Theorem 2.25, Proposition 3.7, Proposition 4.3, Lemma 4.4, Theorem 4.5, and the Corollary 4.7 of the paper are based on these Lemmas. For Lemmas 2.13 and 2.14 by Hila and Dine in [1], the authors refer to Theorem 1 in the paper by Kwon and Lee in [2], but the proof of Theorem 1 in [2] is wrong.

The authors use the usual definition of left (right) regular Γ -semigroups. According to Definition 2.11 of this paper a Γ -semigroup M is called *left regular* if $a \in M\Gamma a\Gamma a$ for every $a \in M$. That is, for every $a \in M$ there exist $x \in M$ and $\gamma, \mu \in \Gamma$ such that $a = x\gamma a\mu a$. According to Definition 2.12, a Γ -semigroup M is called *right regular* if $a \in a\Gamma a\Gamma M$ for every $a \in M$. That is, for every $a \in M$ there exist $x \in M$ and $\gamma, \mu \in \Gamma$ such that $a = a\gamma a\mu x$. In [2] the authors consider the following definition of a Γ -semigroup (shortly *po- Γ -semigroup*): Let M and Γ be two nonempty sets. M is called a Γ -semigroup if

- (1) $M\Gamma M \subseteq M$, $\Gamma M \Gamma \subseteq \Gamma$ and
- (2) $(a\gamma b)\mu c = a(\gamma b\mu)c = a\gamma(b\mu c)$ for all $a, b, c \in M$ and all $\gamma, \mu \in \Gamma$.

Then an ordered Γ -semigroup is a Γ -semigroup endowed with an order relation " \leq ". This is the Theorem 1 of the paper in [2]:

Theorem 1. *Let M be a po- Γ -semigroup. The following are equivalent:*

- (1) M is left regular.
- (2) $L(a) \subseteq L(a\gamma a)$ for every $a \in M$ and every $\gamma \in \Gamma$.
- (3) $a\mathcal{L}(a\gamma a)$ for every $a \in M$ and every $\gamma \in \Gamma$.

And this is the proof of (1) \Rightarrow 2 of Theorem 1 (I copy it from [2]): "Let M be left

regular. If $t \in L(a)$, then $t \leq a$ or $t \leq x\gamma a$ for some $x \in M$ and $\mu, \gamma \in \Gamma$. Since M is left regular, $a \leq y\mu(a\gamma a)$ for some $y \in M$ and $\mu, \gamma \in \Gamma$. If $t \leq a$, then $t \leq a \leq y\mu(a\gamma a)$ ($y \in M, \mu, \gamma \in \Gamma$). If $t \leq x\gamma a$, then $t \leq x\gamma a \leq x\gamma(y\mu a\gamma a) = (x\gamma y)\mu(a\gamma a)$. In any case, $t \leq z\mu(a\gamma a)$ for some $z \in M$. Hence $t \in L(a\gamma a)$, and so $L(a) \subseteq L(a\gamma a)$ ".

The phrase "Since M is left regular, $a \leq y\mu(a\gamma a)$ for some $y \in M$ and $\mu, \gamma \in \Gamma$ " in it is wrong as the γ in $y\mu(a\gamma a)$ cannot be the same with the γ in $x\gamma a$. The correct is "If $t \in L(a)$, then $t \leq a$ or $t \leq x\gamma a$ for some $x \in M$ and $\mu, \gamma \in \Gamma$. Since M is left regular, $a \leq y\mu(a\xi a)$ for some $y \in M$ and $\mu, \xi \in \Gamma$ ". So the proof of the implication (1) \Rightarrow (2) in Theorem 1 in [2] is false, and so is the proof of Theorem 1 given by Kwon and Lee in [2].

This is a copy from the paper in [1; p. 613, lines 5–11]: "As an application of the result proved in Theorem 1 in [Kwon, Y. I, Lee, S. K.: On left regular po - Γ -semigroups. Kangweon-Kyungki Math. Jour. 6 (1998), No. 2, pp. 149–154] (which is the References [2] in the present note) we have the following lemmas:

Lemma 2.13. *Let M be a Γ -semigroup. The following are equivalent:*

- (1) *M is left regular.*
- (2) *$a\mathcal{L}(a\gamma a)$ for every $a \in M$ and every $\gamma \in \Gamma$.*

Lemma 2.14. *Let M be a Γ -semigroup. The following are equivalent:*

- (1) *M is right regular.*
- (2) *$a\mathcal{L}(a\gamma a)$ for every $a \in M$ and every $\gamma \in \Gamma$."*

Although the definition of the Γ -semigroup considered in [2] differs from the definition of a Γ -semigroup considered in [1], if the Theorem by Kwon-Lee were true, then the two Lemmas by Hila-Dine would be true as well. It might be also noted that not only the proof of the Theorem 1 in [2] is wrong, but the characterization of a po - Γ -semigroup in which $a\mathcal{L}(a\gamma a)$ given in [2] is also wrong. The characterization of a Γ -semigroup in which $a\mathcal{L}(a\gamma a)$ (or $a\mathcal{R}(a\gamma a)$) given in [1] is wrong as well.

References

- [1] Hila, K., Dine, J.: On Green's relations, 2^0 -regularity and Quasi-ideals in Γ -semigroups. Acta Math. Sinica, English Series, **29**(3), 609–624 (2013)
- [2] Kwon, Y.I, Lee, S.K.: On left regular po - Γ -semigroups. Kangweon-Kyungki Math. Jour., **6**(2), 149–154 (1998)

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